Pyramidal horn feeds for paraboloidal dishes

by DAIN EVANS, G3RPE*

Introduction

Pyramidal horns have an important advantage over other types of feed for dishes which makes them especially suited for use by amateurs; they offer a virtually perfect match over a wide range of frequencies. Even large dimensional errors need not affect the quality of this match, but only the efficiency of illumination of the dish. By contrast, the frequency of operation of the common dipole/reflector type of feed and, to a lesser degree, that of designs such as the Cutler feed, is critically dependent on their dimensions, and small errors (even hundredths of an inch at the highest frequencies) can lead to quite severe matching problems.

A second advantage of horns is that they can be designed on paper usually with sufficient accuracy to produce optimum illumination of the dish in *both* planes. In the case of dipole feeds there is little control over the beamwidth of the feed, especially in the E-plane, and the overall gain of the aerial usually suffers significantly as a consequence.

The first section of this article presents a straightforward method of designing feeds for dishes of medium and long focal length, and this is followed by examples of practical designs. In later sections two methods are given for designing shaped sub-reflectors which enable horn feeds to be used with dishes of short focal length; this is an alternative approach to the use of dipole and similar feeds with their associated disadvantages.

Basic design data

The characteristics of a perfect paraboloidal dish can be specified by two factors only. The first is its diameter D, using the notation of Fig 1, which determines the potential gain of the aerial at a particular frequency. The second is the ratio of its focal length to its diameter, f/D, which is the fundamental factor determining the design of a suitable feed. This factor is of course related directly to the angle subtended by the rim of the dish at its focus, and therefore also to the beamwidth of the feed necessary to illuminate it efficiently. Two dishes of different diameter but having the same f/D ratio can employ identical feeds. Dishes having the same diameter but different focal lengths can require quite dissimilar feeds. For a perfect dish, f is equal to $D^2/16c$ and f/D to $D/16c$ where c is the depth of the dish at its centre. Practical values for f/D usually range from 0·25 for a focal plane dish up to 1·0 for one of long focal length.

The dimensions of a pyramidal horn feed as a function of the f/D of a dish are given in Fig 2. These were derived using the procedure given in an earlier article [1], and are based on an edge illumination of 10dB down on that at the centre, a value appropriate for amateur usage. The relationship between beamwidth and horn dimensions which was necessary in compiling Fig 2 was obtained from [2]. It is believed that this data was determined experimentally: it was noted in the earlier article that when the aperture of a horn is less than 2λ, as is usually the case with feeds, then simple design theory breaks down.

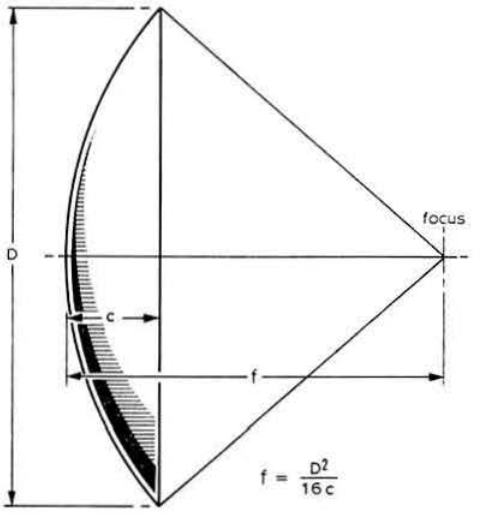

Fig 1. Basic parameters of a paraboloidal dish

The dimensions of the feed are given in terms of wavelength at the frequency of operation, the actual dimensions, of course, being obtained by multiplying the values of A/λ and B/λ by the wavelength in air at the design frequency. The minimum length of the horn L (as defined in Fig 2) must exceed the value A^2/λ; normally this presents no problems. Selected values of λ are given in [3].

It can be seen from Fig 2 that as the f/D ratio of the dish is reduced, so the aperture of the horn decreases. The limit of the present design data is first reached with the A dimension at a value of A/λ equal to 0·8, which corresponds to a minimum f/D ratio of 0·48. Although there are various techniques for reducing the effective aperture of the horn, thereby enabling dishes having a smaller f/D ratio to be properly illuminated, the design of each feed must be determined experimentally, with the added difficulty that the beamwidth becomes very sensitive to small changes in dimensions: for examples see [4] and [5]. A lower limit for the f/D ratio at 0·48 therefore becomes something of a practical limit for the many microwave enthusiasts who do not have access to the appropriate test equipment. However, as shall be seen in a later section, methods are available for increasing the effective focal length of a dish which enables the advantages of horn feeds to be enjoyed even with dishes of short focal length.

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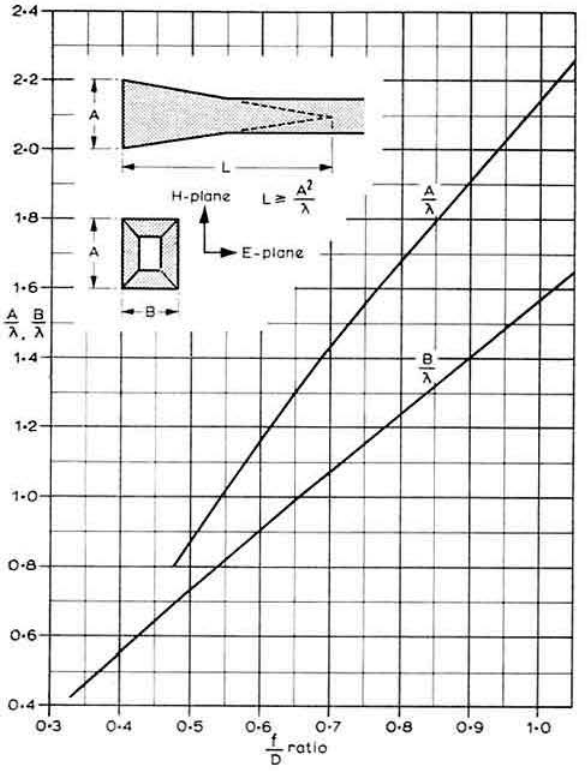

Fig 2. Relationship between horn feed dimensions and focal length/diameter ratio of a dish

Practical horn designs

A distinction must be drawn between the higher and lower microwave bands. At 24GHz, 10GHz and probably 6GHz, the horn will normally be fed by a waveguide, since the loss of even high-quality coaxial cable is significant. The horn feed will therefore consist of a length of waveguide, almost certainly of a standard size which matches that used in the rest of the system, the open end being flared in one or two dimensions to produce the beamwidth of radiation necessary to illuminate the dish efficiently. On the lower bands, 1,296MHz, 2,304MHz and 3,456MHz, the feed will almost certainly be fed via coaxial cable rather than waveguide. The horn in this case will therefore include a waveguide/coaxial cable transition in its design. The waveguide may be of a standard size (but need not be) and advantage of this may be taken to facilitate the construction of the transition. This will be covered in a later article.

As an example of the design of a horn for a particular dish, take a dish of diameter $D = 36$in, of depth $c = 4{\cdot}26$in for use at 10,050MHz for which $\lambda = 1{\cdot}174$in. The focal length f is given by $D^2/16c = 19{\cdot}0$in and the f/D ratio is therefore 0·53. From Fig 2 the following values are obtained:

H-plane aperture $A/\lambda = 0{\cdot}96$; $A = 0{\cdot}96 \times 1{\cdot}174 = 1{\cdot}127$in
E-plane aperture $B/\lambda = 0{\cdot}78$; $B = 0{\cdot}78 \times 1{\cdot}174 = 0{\cdot}916$in
and $L \geq A^2/\lambda = 1{\cdot}08$in.

At this frequency, a suitable waveguide is No 16, [3]. The practical configuration of this design of horn is shown in Fig 3. Note that the length of the taper greatly exceeds the minimum value of L specified above.

By the same process of design, a horn feed for any dish of the same f/D ratio but for 1,296MHz (for which $\lambda = 9{\cdot}10$in) would have an aperture of 8·74 by 7·10in, tapering to 6·5 by 3·25in if WG6 were used, and one for 24GHz (for which $\lambda = 0{\cdot}4918$in) would have an aperture of 0·472 by 0·384in, tapering to 0·420 by 0·170in (WG20).

There is a special case of horn design in which the A dimension is made equal to the broad internal width of the waveguide. This means that the horn need be flared only in one direction, which considerably simplifies its manufacture. However, such a horn can only be used with a dish of a particular f/D ratio. For example, the A dimension of WG16 is 0·9in, which is equivalent to an A/λ at 10,368MHz of 0·79. Reference to Fig 2 shows this value to correspond to an f/D ratio of 0·48.

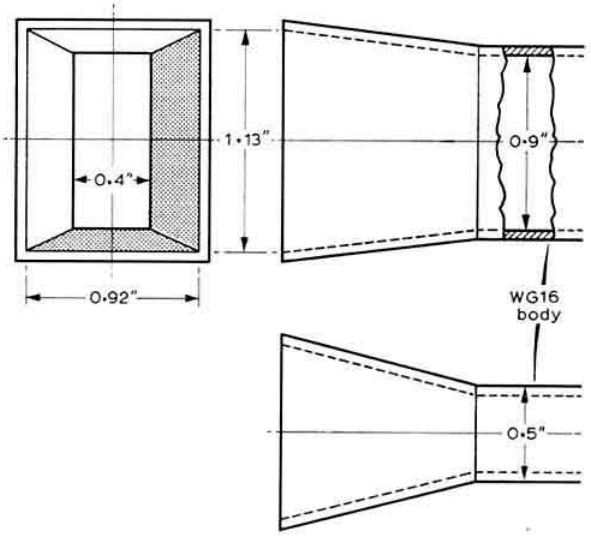

Fig 3. Horn design for use at 10,050MHz with a dish of f/D ratio of 0·53

Dish/feed arrangements

Direct feed

This common method of feeding a dish is shown schematically in Fig 4. The horn lies along the axis of the dish and points directly at its centre. The only adjustment necessary is the distance from the horn to the dish; the horn is therefore mounted in a bearing which is supported from either the dish or its support structure. A practical version is shown in Photo 1.

If the phase centre of the horn, that is the point source from which radiation is assumed to emanate, were accurately known, then it could be made to coincide precisely with the focus; since it is not, the position of the horn is made adjustable within perhaps ± 5 per cent of the focal length of the dish. An approximate position of the horn can be obtained by making a scale drawing of the system and by arranging that the end of the horn intercepts lines drawn from the focus to the rim of the dish, as shown in Fig 4.

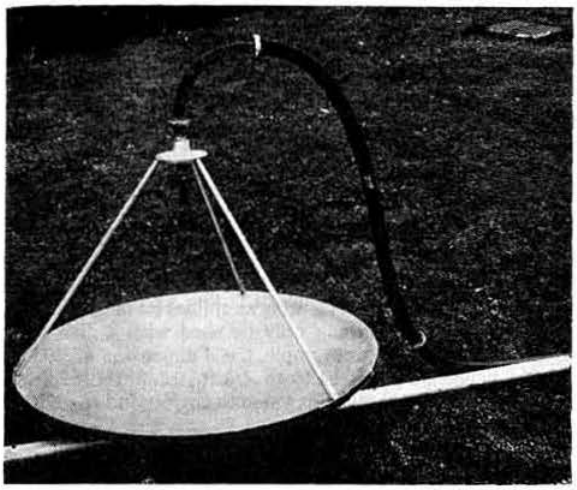

Photo 1. Practical version of direct feed arrangement

The main advantages of this method of mounting the feed are its simplicity in conception and construction, its high efficiency, and also that it leaves the back of the dish clear. Mounting a dish accurately with respect to its support structure is obviously simpler if it is mounted symmetrically, and with high-gain aerials having a beamwidth of perhaps a degree or two this becomes a significant advantage.

The main disadvantage of this method is the bulkiness of the feed support structure with dishes of long focal length. However, if the structure is well engineered it should be possible to demount it and, after transportation, re-assemble it without affecting its alignment. If equipment is to be mounted behind the dish and fed by waveguide, an awkward run is required.

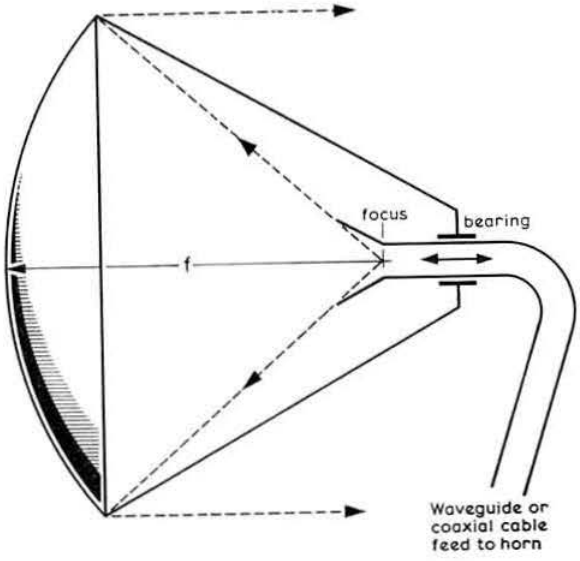

Fig 4. Schematic form of direct feed arrangement

Rear feed

This method is shown schematically in Fig 5. The horn is mounted from a bearing in the centre of the dish, and radiation is reflected by a sub-reflector back on to the dish. The phase centre of the feed is placed at the virtual focus of the dish, which is the real focus as reflected by the sub-reflector. For dishes of medium and long focal length, which is the case considered in this section, a plane sub-reflector can be used. Under these conditions, the horn "sees" a virtual dish having the same f/D ratio as the real dish, and therefore the design of the horn feed is done using this ratio as described earlier.

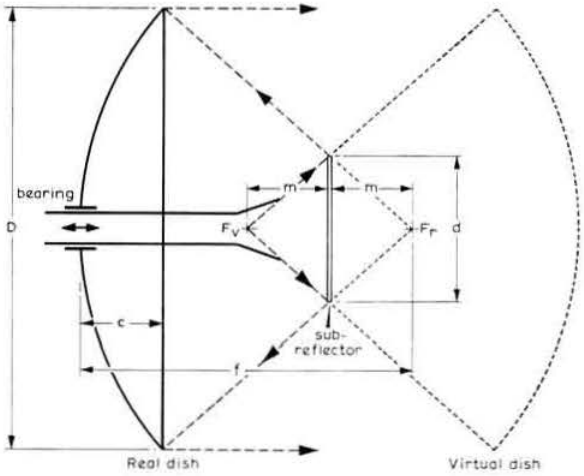

Fig 5. Construction used in designing a rear feed system

The diameter of the sub-reflector should preferably exceed 5λ in diameter, at which value the loss in gain due to diffraction around the rim will be less than 0·5dB. However, when small dishes are used at lower frequencies this size may be impractical. Typical diameters used range from a third of the diameter of the dish for small dishes to a tenth for large dishes. If anything, the reflector can be made on the large side as its effect in obscuring the dish is surprisingly small—about 1dB loss for a reflector/dish diameter ratio of 0·3.

Once the diameter of the sub-reflector is fixed, then its position is also fixed; the rim of the reflector must intercept lines drawn from the real focus F_R to the rim of the dish. Its position can be calculated from the relationship:

$$m = \frac{d}{D}(f - c)$$

using the notation of Fig 5.

The position of the virtual focus F_V is the same distance in front of the reflector as the real focus is behind. F_V then defines the position of the phase centre of the horn feed.

The rules given above presume that optical theory is obeyed precisely, which is not quite the case. For reasons given earlier it will be necessary to adjust the position of the horn relative to the reflector. It may also be necessary to move the reflector closer to the dish by up to $\lambda/4$, and the diameter of the reflector should preferably be made slightly oversize to ensure that it still intercepts lines drawn from the real focus

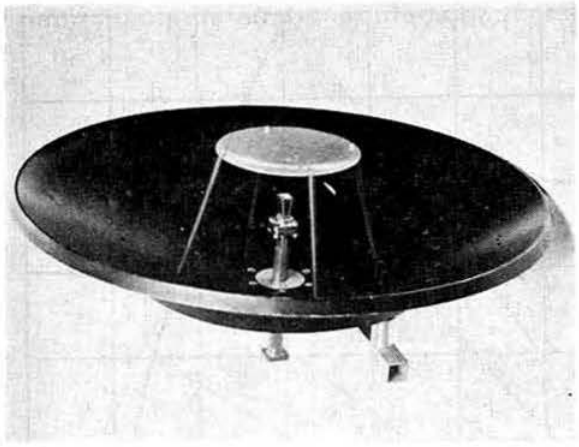

Photo 2. Practical version of rear feed arrangement

to the rim of the dish when in this position. It is well worthwhile making a scale drawing of the proposed aerial system, both to help in the basic design and also to avoid the easily overlooked factors, such as the positioning of flanges, which may affect the range of adjustment of the horn and reflector.

A common method of mounting the reflector is from support rods bolted to the dish. As these rods are in the strong field of the aerial, they should preferably be made from an insulator such as Tufnul. If the support rods are made equal in length, and mounted symmetrically on both reflector and dish, then the assembly is to a large degree self-aligning. An example of this construction is shown in Photo 2. A practical tip is to use four support rods rather than three; if all four fit accurately, then it will be almost certain that the reflector is well aligned.

With the above method, the position of the reflector is fixed and independent of adjustments to the feed. A second method is to mount the reflector from a sliding bearing on the waveguide run to the horn. In this case the position of the reflector will have to be adjusted as the feed is moved in and out in order to maintain its position correctly with respect to the dish. This process becomes less critical if the reflector is made slightly larger than strictly necessary.

The main advantage of this method is that the support structure is less bulky than when the direct feed is used, and that the rear feed is more convenient if waveguide equipment is to be fitted behind the dish. If the reflector is removed, then the horn itself may be used as a low-gain broad-beamwidth aerial. The disadvantages of the rear-feed arrangement are that the dish support structure must be designed to avoid the feed, and an extra component—the sub-reflector—needs to be aligned accurately, which complicates construction and setting up.

Horn feeds for short focal length dishes

The methods given above for the design of simple aerial systems are limited to dishes having an f/D ratio exceeding 0·48, for the mundane reason that systematic design data for the beamwidth of very small horns are apparently not available. A method of avoiding this limitation, at the expense of some complication, is described below.

It was pointed out in the previous section describing the rear feed method that the horn "sees" a virtual dish which, because the reflector is plane, has the same f/D ratio as the real dish. If, however, the reflector is suitably shaped, the virtual dish the horn "sees" can be made to have a focal length either greater or smaller than the real dish. In the Cassegrain system, the reflector is a hyperboloid, and in the Gregorian system, elliptically shaped. In this article only the Cassegrain system will be considered as the Gregorian appears to offer no special advantages.

Two methods are given below for determining the precise shape of the reflector required. The first is an elegant graphical method due to G3HWR, which is a recommended first step even if the second method is preferred, that of calculating the profile of the reflector directly. The second method produces results in a form specially suited for machining the reflector from solid material. Both methods will be described as a series of steps using a worked example as a guide. If a calculator is used, par time for both methods is about 30min.

Graphical method

The notation to be used is given in Fig 6. Suffixes of r or v are used to distinguish parameters associated with the real and virtual dishes respectively. The completed construction, actually for an 18in dish intended for 24GHz operation, is shown in Fig 7.

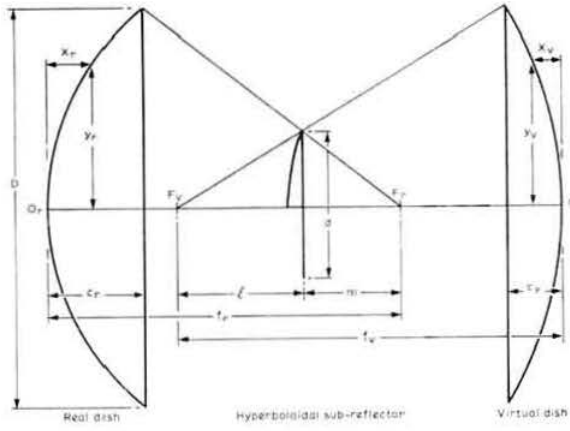

Fig 6. Notation used in the construction of hyperbolic sub-reflectors

(1). Measure the diameter of the dish D = 18in, and its depth c_r = 3·673in.
Then $f_r = D^2/16c_r$ = 5·513in and f_r/D = 0·3063, ie well below the minimum value 0·48 for direct feeding by a horn.
(2). Calculate the profile of the dish for a reasonably large number of values from:

$$x_r = y_r^2/4f_r = 0{\cdot}04535y_r^2$$

$\pm y_r$(in)	x_r(in)	$\pm y_r$(in)	x_r(in)
0	0	5	1·13
1	0·04	6	1·63
2	0·18	7	2·22
3	0·41	8	2·90
4	0·73	9	3·67

The value of x_r corresponding to y_r = 9in should, of course, equal c_r. Plot the profile on squared paper, as shown in Fig 7.

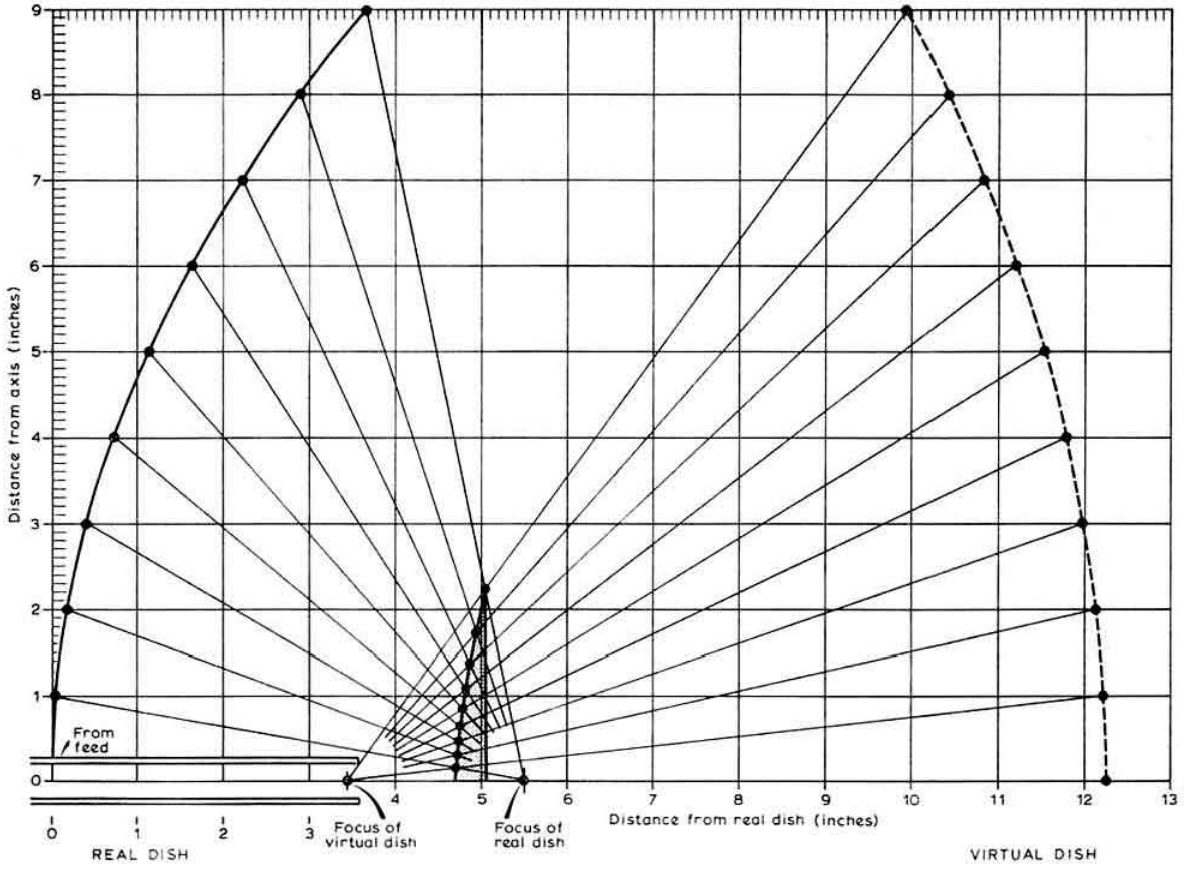

Fig 7. Example of graphical construction of hyperbolic sub-reflector

This profile may be transferred to card and used to check the accuracy of the dish, as well as the measurements and calculations.

(3). Select the ratio f_v/D for the virtual dish, the diameter of the reflector d and the position of the horn feed within the limits:

(a) The ratio f_v/D must exceed 0·48. Consider using preferred values corresponding to horns flared only in one dimension. No obvious upper limit.

(b) Diameter of reflector d: not critical, typical values of d/D being 0·1 for large dishes, up to 0·3 for small dishes.

(c) Position of feed: again not critical. The feed may even be fitted behind the dish, firing through a suitable hole, if more convenient. Allowance must be made for its adjustment.

When two of these three parameters are specified, then the third is defined.

In this example a horn feed for 24GHz based on WG20, and taking advantage of the special case where the horn is flared in the B dimension only, will be designed. Thus the A dimension of the horn will be the internal width of the waveguide, 0·420in, and $A/\lambda = 0{\cdot}420/0{\cdot}4918 = 0{\cdot}85$. The f/D ratio corresponding to this value is obtained from Fig 2: 0·49. As the virtual dish has the same diameter as the real dish,

$$\text{then } f_v = 0{\cdot}49 \times 18{\cdot}00 = 8{\cdot}820\text{in},$$
$$\text{and } c_v = D^2/16f_v = 2{\cdot}296\text{in}.$$

A reflector diameter d equal to 4·5in, which corresponds to a d/D ratio of 0·25, is selected. Then, in the notation of Fig 6,

$$m = \frac{d}{D}(f_r - c_r) = 0{\cdot}460\text{in}$$
$$\text{and } l = \frac{d}{D}(f_v - c_v) = 1{\cdot}631\text{in}$$

which values define the position of the reflector and feed point with respect to the real focus F_r.

(4). The origin of the virtual dish O_v with respect to the real dish O_r is given by $f_r + f_v - l - m = 12{\cdot}24$in.

(5). Using the same values of y as in (2), calculate the profile of the virtual dish from the equation

$$x_v = y_v^2/4f_v = 0{\cdot}02834y_v^2.$$

Plotting the curve is easier if the values of x_v are subtracted from the value for O_v determined in (4):

$\pm y_v$(in)	x_v(in)	12·24 − x_v(in)	$\pm y_v$(in)	x_v(in)	12·24 − x_v(in)
0	0	12·24	5	0·71	11·53
1	0·03	12·21	6	1·02	11·22
2	0·11	12·13	7	1·39	10·85
3	0·26	11·98	8	1·81	10·43
4	0·45	11·79	9	2·30	9·94

The value of x_v corresponding to $y_v = 9$ should, of course, equal c_v.
(6). Join each of the plotted points for both dishes to their respective foci. The points of intersection of lines corresponding to the same values of y_r and y_v then define the required hyperbola.
(7). Calculate the dimensions of the horn from the A/λ and B/λ values given in Fig 2 corresponding to the ratio f_v/D of the *virtual* dish. In the example the A dimension was originally specified as 0·420in; the B dimension = 0·72 × 0·4918 = 0·354in. The position of the horn can be determined approximately by drawing it as in Fig 7, so that the end of the horn intercepts the line drawn from the focus F_v to the rim of the virtual dish.

It is to be noted that while the construction given above refers to a particular set of parameters, it is rather more general in application than at first sight. First, if all the linear dimensions are scaled by a constant factor, then the geometry obviously remains unchanged; the construction is identical in form to all others having the same f_r/D, f_v/D and d/D ratios. Second, the same construction also applies to any feed for any other frequency which is suitable for illuminating a virtual dish having the f/D ratio specified.

Not quite so obvious is that this particular construction holds for a range of related values of f_r/D, f_v/D and d/D. If, for example, the parts of the construction corresponding to y_r, $y_v = 9$ are ignored, the same hyperbola will be generated, although smaller in size. It may be practical, therefore, to generate quite general design information rather more simply than one first supposes.

Mathematical method

This uses the notation given in Fig 6.

(1). Select or calculate values of f_r, f_v, d, l and m as described above in steps (1) and (3).
(2). The equation of a hyperbola is

$$x^2 = a^2 + y^2/(e^2 - 1)$$

where x is measured from the midpoint between the real and virtual foci, and

$$a = \frac{(l + m)(f_v - f_r)}{2(f_v + f_r)} = 0{\cdot}2412\text{in},$$

$$\text{and} \quad e = \frac{(l + m)}{2a} = 4{\cdot}334.$$

The equation of the hyperbola required is therefore:

$$x^2 = 0{\cdot}05819 + 0{\cdot}05623y^2$$

from which the following values may be calculated:

±y(in)	x(in)	x − 0·241(in)	±y(in)	x(in)	x − 0·241(in)
0	0·241	0	1·250	0·382	0·141
0·250	0·248	0·007	1·500	0·430	0·189
0·500	0·269	0·028	1·750	0·480	0·239
0·750	0·300	0·059	2·000	0·532	0·291
1·000	0·338	0·097	2·250	0·586	0·345

By subtracting the "zero" value 0·241 from each of the values of x, the x and y co-ordinates are in a form more suitable for plotting profiles for checking potential reflectors, or for machining a reflector from solid.

Practical reflectors

Both the graphical and mathematical methods for determining the shape of the reflector give data which are very precise in terms of a wavelength at amateur operating frequencies. Reflectors machined to these dimensions can readily be made accurate to within a few thousandths of an inch, which represents a maximum error in the region of $\lambda/100$ even at the highest frequencies at present available. Clearly, such an error can be ignored, and the opportunity of having at least one part of the equipment virtually perfect should not be missed.

The question arises of how close the shape of the reflector must be to that of a perfect hyperboloid. A general rule applicable to dishes is that the peak error should be less than $\lambda/10$ at the operating frequency. Perhaps a good guess for sub-reflectors is to reduce this value by the ratio of the real and virtual dishes, f_v/f_r, which factor may range from about 1·5 to 4. On this basis, if two curves are drawn parallel to the hyperbola as developed and spaced by this fraction of a wavelength, then any reflector having a profile which fits between these curves will be suitable. Obviously there will be a greater freedom of choice at lower frequencies.

The material used to construct the reflector need not be solid. Wire mesh can be used provided that the maximum dimension of the holes is less than $\lambda/10$. Alternatively, the reflector can be built up from wires bent to the correct shape, provided that the gap between them is again less than $\lambda/10$. Note that the shape of the wires remains the same even if positioned away from the axis of the reflector: only one former to shape the wires is required.

References

[1] *Radio Communication* April 1972, p229.
[2] *Reference data for engineers* 5th ed, 1972, pp25, 37.
[3] *Radio Communication* December 1973, p858.
[4] *Radio Communication* July 1971, p467.
[5] *MIT Radiation Laboratory Series*, Vol 12, p381.

NEW BOOK

"Electronics", by L. W. Owers, CEng, MIERE

Sub-titled *An elementary introduction for beginners*, this book is intended to provide an introduction to basic electronics for those meeting the subject for the first time. The approach is essentially non-mathematical and diagrams are extensively employed. Where formulae and units are introduced, SI units are employed. The book provides an explanation of many basic items including atoms, energy, waves, insulators, semiconductors, types of components and their applications. Valves and semiconductors and their applications to radio and television are also covered.

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